Anisotropic dynamics in a shaken granular dimer gas experiment

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The dynamics, velocity fluctuations, and particle-plate interactions for a two-dimensional granular gas of shaken, nonspherical particles are studied experimentally. The experiment consists of a horizontal plate that is vertically oscillated to drive the dynamics of macroscopic dimers, spherical pairs that are loosely connected by a rod that couple the interaction each of the spheres has with the shaking plate. The extended nature of the particles results in more than one energy-momentum transfer between the plate and each dimer per shaking cycle. This complex interaction results in anisotropic behavior for the dimer that is a function of the shaking parameters.

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Recently, a different driven granular gas experiment comprised of two layers of separate species of granular material has demonstrated robust velocity fluctuations that are nearly Gaussian over a wide range of parameters [1]. In the experiment, a horizontal plate vertically drives a first layer of one species of particles whose collisions, in turn, fluidize a second layer comprised of a different species of particles. While the velocity fluctuations in the first layer, driven by the plate, remain strongly non-Gaussian and vary with the driving parameters, the second layer, thermalized by collisions with the first layer, demonstrate nearly Gaussian velocity statistics over a wide range of shaking parameters. Similar to this mechanically fluidized bed, a gas fluidized bed system used to thermalize the motion of a larger sphere also demonstrates nearly Maxwell-Boltzmann statistics for the indirectly driven particle [2]. Clearly, the way in which energy is injected into each layer in the mechanically fluidized experiment is an important detail that must be examined more closely to understand how each layer could simultaneously demonstrate drastically different velocity fluctuations.

In this report, we detail the injection of energy between the shaking plate and the species that comprise the lower layer of the two-layer experiment: granular dimers, spherical pairs loosely connected by a short rod. Experiments to probe the dynamics of driven granular gases, collections of large numbers of dissipative macroscopic particles whose motion is maintained through various forms of external driving, have focused largely on species of identical spheres [3–8]. However, identical spheres represent only one special species of macroscopic granular particles. We demonstrate that many of the differences in the dynamic behavior from that of identical spheres can be understood through anisotropies that directly relate to the geometry of the extended particles. In particular, the manner in which the particles are thermalized by the shaking plate in the vertical direction is intimately connected to how the horizontal dynamics are manifested in the plane of the two-dimensional (2D) granular gas. A prior simulation has investigated developing a kinetic theory for such gases in a freely cooling case [9].

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The experiment consists of an aluminum plate of radius 14.6 cm that is vertically oscillated by an electromagnetic shaker. The plate is level and flat, and the acceleration of the plate, $\Gamma = A(2\pi\nu)^2$, shaking with peak amplitude A, at frequency ν, is uniform, having a spatial variance across the surface that is $<1\%$. The particles are comprised of two hollow spheres that are each 3.2 mm diam and are loosely connected by a thin rod, which allows a spacing between the spheres of zero to 1.6 mm. The total length of the dimer pair can therefore vary from 6.4 to 8.0 mm. Except at the highest densities, where excluded volume effects compress the dimers along the interconnecting rod, the motion of the bouncing on the plate is observed to keep the dimers fully extended.

The dimers are constructed by cutting pairs from chains similar to those used in prior investigations of the motion of chains [10–12]. The loose connection of the rod between the two spheres of the dimer allows for an additional degree of movement where the spheres may rotate about the ends of the rod as wheels on an axle, letting the dimer roll in a direction perpendicular to the connecting rod.

The average mass of a single dimer is ~185 mg but can vary up to ±7 mg from dimer to dimer. The aspect ratio of the dimer, which is the ratio of the length (as measured along the direction of the rod connecting the spheres) to the width, varies from 2 to 2.5, depending on whether the dimer is fully compressed or fully extended. A coefficient of restitution is determined experimentally for the dimers to be ~0.3 [13]. For comparison, the identical technique yields a value of 0.5 for one of the single hollow balls from a dimer. A Delrin ball is measured with the same technique to have a value of 0.9.

The rod connecting the two spheres is a natural geometric axis for the purpose of describing the motion of the particles. When viewed from above, the two-dimensional motion of the dimer in the horizontal plane can be considered in terms of two angles, as shown in Fig. 1. Because of the extended nature of the particle, the dynamics can be decomposed into the motion of the center of mass θ and $\mathbf{\Delta r}$, and the motion about the center of mass ϕ.

The motion of a single dimer on a vertically shaken horizontal plate has been measured using high-speed digital photography [14]. By overlaying two sequential images, a com-

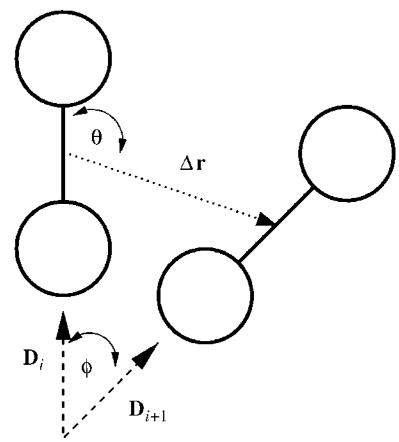

FIG. 1. When viewed from above, the horizontal dynamics of the nonspherical particles are described by two angles. The angular displacement of the center of mass for the particle moving $\mathbf{\Delta r}$ relative to the natural director of the particle made by the connecting rod is θ. The motion about the center of mass of the particle ϕ denotes how the orientation of the director moves relative to a fixed frame.

posite picture similar to Fig. 1 can be produced. The locations of the two spheres that comprise the dimer can be identified using software analysis [15]. Identifying the location of the spheres that constitute the dimer in each frame allows the center of mass and the orientation of the dimer to be determined. If the orientation of the dimer relative to a fixed frame (such as that of the camera) is $\mathbf{D}_i$ and $\mathbf{D}_{i+1}$ in frame i and $i+1$, respectively, then the angular displacement about the center of mass ϕ is determined from the following relationship:

$$\sin(\phi) = \frac{\mathbf{D}_i \times \mathbf{D}_{i+1}}{|\mathbf{D}_i||\mathbf{D}_{i+1}|}. \qquad (1)$$

In a similar fashion, the direction of the motion of the center of mass can be determined by examining the motion of the center of mass $\mathbf{\Delta r}$ from one frame to the next relative to the director of the dimer $\mathbf{D}_i$

$$\cos(\theta) = \frac{\mathbf{D}_i \cdot \mathbf{\Delta r}}{|\mathbf{D}_i||\mathbf{\Delta r}|}. \qquad (2)$$

Probability distributions of $\cos(\theta)$ for a single dimer on a plate for different shaking frequencies at a peak plate acceleration of $2g$ are shown in Fig. 2. The particle was tracked from above with a high-speed camera, and the results demonstrate a preference for the center of mass of the dimer to move along the direction of the rod connecting the two spheres at low frequencies. The reader should note the change in this bias by the decrease in the probability at $\cos(\theta)=\pm 1$ as the shaking frequency is increased. There is an interesting exchange between the motion of and about the center of mass as the frequency is increased. At 50 Hz, when the motion about the center of mass is biased toward the ends, the center-of-mass motion demonstrates Gaussian velocity statistics. As the frequency of shaking is increased to 70 Hz and the motion about the center of mass becomes more uniform, the velocity statistics for the center of mass become slightly non-Gaussian. At 90 Hz, where the motion about the center of mass is now biased for motion perpendicular to the director, the velocity statistics for the center of mass motion is strongly non-Gaussian. To understand this anisotropy in the dimer motion, it is necessary to examine how the single dimer interacts with the vertically shaking plate.

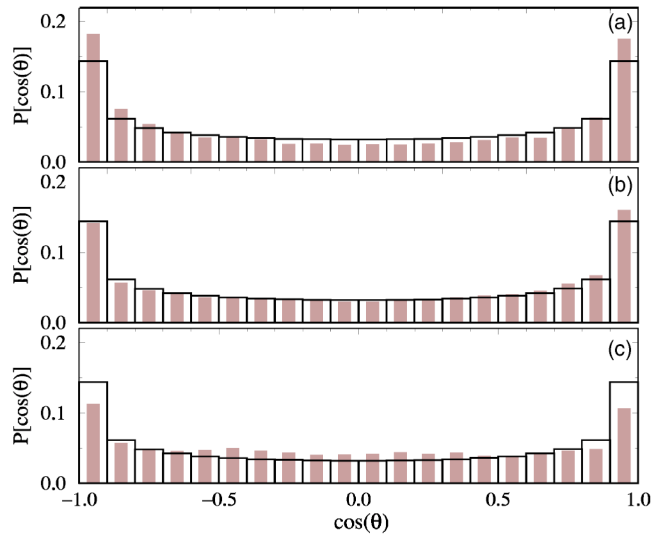

FIG. 2. (Color online) Histograms (dark boxes) for $\cos(\theta)$, the direction of center of mass motion of a single dimer on a plate with a peak acceleration of $2g$ for (a) 50 Hz, (b) 70 Hz, and (c) 90 Hz. The distribution for $\cos(\theta)$ for a uniform distribution of θ is shown by the open boxes in each figure. The distribution demonstrates a bias for the dimer to move in a direction along the director rod joining the two spheres of the dimer at low frequency. The bias is shifted to motion perpendicular to the connecting rod as the shaking amplitude is decreased (shaking frequency is increased).

Figure 3 is an example of the signal acquired from the accelerometer to measure the peak acceleration of the shaking plate in this experiment. The data shown is for one typical oscillation of the plate with a peak acceleration of $2g$ at a frequency of 50 Hz. The signal is comprised of two parts. The sinusoidal wave is the measured acceleration of the plate. The two "noisy" portions of the signal (one near the trough and one near the peak of the oscillation) are the pings caused by each of the balls of the dimer colliding with the plate. The trough (negative acceleration) ping corresponds to a ball of the dimer colliding with the plate as it accelerates downward [see also Fig. 4(a)]. The peak ping corresponds to the other ball of the dimer colliding with the plate as it accelerates upward [Fig. 4(c)]. The bias in Fig. 2 can be understood in light of this dimer-plate interaction in the vertical direction in terms of the individual collisions of each sphere of the dimer.

The motion of a single sphere interacting with a shaking plate has been studied previously [16]. In carefully examining the motion of the single sphere throughout the shaking cycle, regions of the system's phase space were observed to either increase or decrease the sphere's momentum. These regions were deemed transmitting and absorbing, respectively [16], and were responsible for the general "chattering"

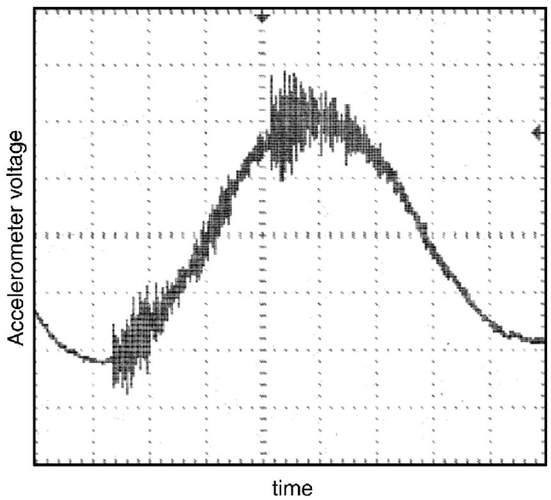

FIG. 3. Accelerometer signal for a single dimer chattering on the plate. The vertical scale is 100 mv ($1g$) per major division. The horizontal scale is 2.5 ms per major division. The driving frequency was 50 Hz, and the acceleration amplitude was $2g$.

up and down in the amplitude of the sphere's motion over several shaking cycles. The analysis for the motion of the dimer may be interpreted for two spheres colliding at nearly opposite phases of the plate cycle.

When the one ball of the dimer collides with the plate during its downward acceleration, the dimer loses momentum (as would a single sphere in that phase of the shaking cycle [16]) in the direction along the rod [Fig. 4(a)]. The first ball of the dimer recoils from this "soft" collision with the plate with a reduced velocity as the other ball of the dimer, under the influence of gravity, begins to fall toward the plate [Fig. 4(b)]. When the plate is accelerating upward, the dimer gains a momentum kick from the second sphere colliding with the plate [Fig. 4(b)], as would a single sphere under the same circumstances [16]. That momentum kick is transmitted up the loose connection of the rod of the dimer. The result is a net momentum transfer along the rod of the dimer. This net interaction with the shaking plate results in a bias of the particle to move in a direction along the geometric axis of the dimer. The biasing effect grows at fixed acceleration as the frequency is decreased because the amplitude of shaking increases, increasing the vertical angle the dimer makes relative to the horizontal plane and emphasizing the bias between the two collisions. As the amplitude of shaking decreases, this angular rotation decreases and the effect is minimized but never completely disappears (see Fig. 2). It should also be noted that this net momentum transfer led to a second important difference from the dynamics of monomers [3,17]. In the monomer case, clustering could be avoided within the system by shaking at an acceleration as little as $1.25g$ for some values of A and ν. Because of the more complicated momentum transfer between the dimers and the plate, accelerations of $\approx 4g$ were necessary to produce a nearly uniform density gas of dimers, i.e., to avoid clustering.

For more than one dimer on the plate, the accelerometer signal becomes more complicated and it is difficult to determine which particle is interacting with the plate. However, in a future paper we will demonstrate a method of describing the net momentum transfer for several dimers interacting with the plate to show that collisions occur more uniformly throughout the shaking cycle than in the monomer case. Our goal in the current work is to describe the velocity statistics of an inelastic granular gas comprised of many dimers (800–3000) as measured via high-speed photography from above the plate for motion in the horizontal plane. The results for some parameters are shown in Fig. 5. In nearly all regimes, the velocity distributions demonstrate non-Gaussian behavior similar to that of the monomer experiment of Olafsen and Urbach [3,17]. It is interesting to note, however, that at the lower density, the velocity distribution function is nearly Gaussian. This can also be understood in light of the complex interaction the particles have with the plate.

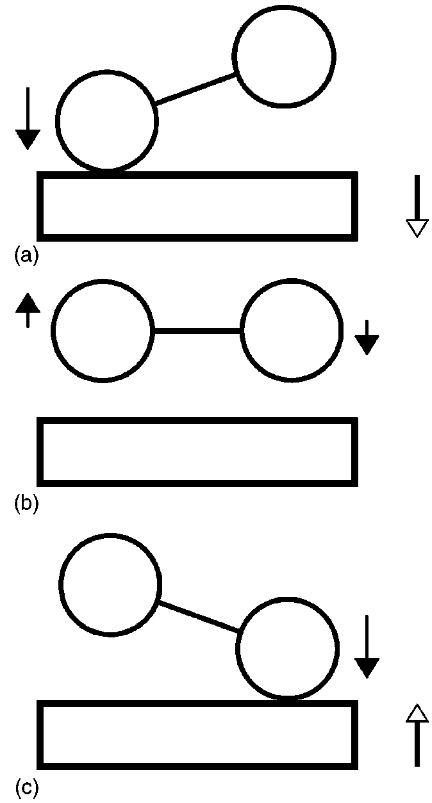

FIG. 4. Viewed from the side, the dimer's motion is biased by the relative motion of the shaking plate during each collision. In (a), the one sphere of the dimer collides while the plate is accelerating downward. In this collision, the plate absorbs some of the momentum of the ball that recoils with a slower velocity as the second sphere begins to fall toward the plate (b). In (c), the second sphere of the dimer collides while the plate is accelerating upward, this collision imparting a momentum gain to the dimer.

The density of the dimer gas, or coverage of the plate, is calculated in terms of the equivalent number of dimer balls that would be needed to have a single layer on the shaking plate. This number is 7320 spheres or 3660 dimers (assuming full compression of all dimers along their connecting rods for one full layer). The density is then calculated as a fraction of the number of dimers in the cell normalized by 3660.

At low densities, the dimers do not undergo as many collisons with the other particles in the gas and the horizontal

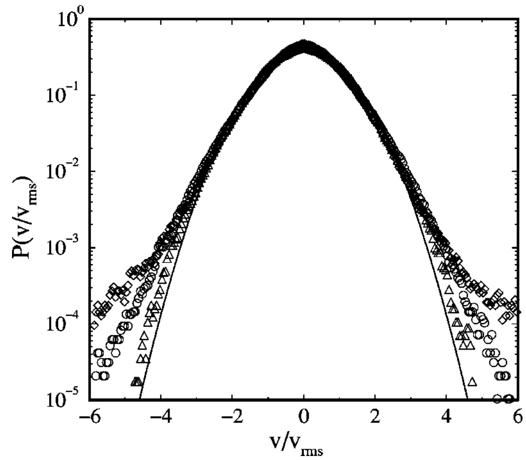

FIG. 5. Velocity fluctuations for different shaking parameters. Triangles: Frequency is 40 Hz, $\Gamma = 1.65g$, density=0.27, $v_{rms} = 1.74$ cm/s. Circles: Frequency is 65 Hz, $\Gamma = 2.0g$, density=0.87, $v_{rms} = 1.04$ cm/s. Diamonds: Frequency is 65 Hz, $\Gamma = 1.5g$, density =0.55, $v_{rms} = 0.94$ cm/s.

motion is dominated by local surface roughness of the plate, resulting in the nearly Gaussian distributions in the horizontal direction. However, the dimers still interact, so the motion cannot completely be thought of as that of a single isolated dimer as was the case in Fig. 2. Indeed, at higher densities, the horizontal motion is dominated by dimer-dimer collisions, which, in general, can be thought of as belonging to one of three types. Because each dimer has two interactions with the shaking plate as previously described, each dimer can be thought of as a pair of loosely coupled spheres, one for which there was a net momentum gain p_+, and one for which there was a net momentum loss p_- in its most recent interaction with the shaking plate.

Collisions between any two dimers i and j are therefore interactions of their constituent spheres that have most recently both received positive momentum kicks $\langle p_+^i p_+^j \rangle$, both received negative momentum kicks $\langle p_-^i p_-^j \rangle$, or two spheres that have received one of each type of kick from interacting with the plate, $\langle p_+^i p_-^j \rangle$ or $\langle p_-^i p_+^j \rangle$. The way in which energy and momentum are transferred from the vertical to the horizontal is therefore much more complicated to model. In general, as the density of the dimer gas increases, the velocity statistics become more non-Gaussian. However, because of the additional complexity of dimer collisions, it is not surprising that the overall shape of the deviations from Guassian statistics do not mimic the monomer case, where the shape of the entire distribution could be "tuned" from nearly Gaussian to nearly exponential, depending on the shaking parameters and how "two-dimensional" the system was constrained to be [17]. The other general trend observed in the data is that the deviation from Gaussian behavior increases as the root-mean-square velocity of the inelastic gas decreases.

In conclusion, in this paper we detail similarities and differences in the observed dynamic behavior of a driven granular gas composed of dimers (linked spheres) from that of granular gases with monomer species. The dynamic differences are directly related to the geometric anisotropies of the constituent particles and the additional complexity with which these particles interact with the shaking plate. The results clearly underscore the need to better understand how energy and momentum are injected into such systems in order to study the dynamics of driven granular gases.

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